Radiative Corrections to Democratic Lepton Mixing

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Abstract

A new ansatz of democratic lepton mixing is proposed at the GUT scale and the radiative corrections to its phenomenological consequences are calculated at the electroweak scale. We demonstrate that it is possible to obtain the experimentally favored results for both neutrino masses and lepton flavor mixing angles from this ansatz, provided the neutrino Yukawa coupling matrix takes a specific nontrivial pattern. The seesaw threshold effects play a significant role in the running of relevant physical quantities.

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To interpret the small neutrino mass-squared differences and the large lepton flavor mixing angles observed in solar and atmospheric neutrino oscillation experiments [1]–[4], a lot of models or ansätze of lepton mass matrices have been proposed at low-energy scales [5]. Among them, the scenarios based on the leptonic flavor democracy and its explicit breaking [6]–[8] are particularly simple, suggestive and predictive. Such a phenomenologically interesting picture might only serve as the low-scale approximation of a more fundamental model responsible for the origin of lepton masses and flavor mixing at a superhigh energy scale (e.g., the grand unified theory (GUT) scale $\Lambda_{\rm GUT} \sim 10^{16}$ GeV). In this case, the renormalization effects between high and low scales have to be taken into account, because they are likely to modify the neutrino mass spectrum and lepton flavor mixing parameters in a significant way [9].

Possible radiative corrections to the ansätze of democratic lepton mixing [6], which can naturally arise from the slight breaking of $S(3)_L \times S(3)_R$ flavor symmetry (i.e., flavor democracy) of the charged lepton mass matrix and that of $S(3)$ flavor symmetry of the effective Majorana neutrino mass matrix, have been discussed between the typical seesaw scale ($\Lambda_{\rm SS} \sim 10^{13}$ GeV) and the electroweak scale ($\Lambda_{\rm EW} \sim 10^2$ GeV) in Ref. [10] [2]. It is found that the mixing angle responsible for the atmospheric neutrino oscillations, defined as θ_{23} in the standard parametrization of the 3×3 lepton flavor mixing matrix [11], is rather insensitive to the renormalization effect. Hence it is very difficult to achieve the experimentally favored result $\theta_{23} \approx 45^\circ$ at $\Lambda_{\rm EW}$ from the model prediction $\theta_{23} \approx 54^\circ$ [6] at $\Lambda_{\rm SS}$. One possible way out is to prescribe a similar ansatz of lepton mass matrices above the seesaw scale; e.g., at or close to the GUT scale. Then the radiative corrections from $\Lambda_{\rm GUT}$ to $\Lambda_{\rm EW}$ will include the seesaw threshold effects, which come from integrating out the heavy right-handed neutrinos step by step at their mass thresholds M_i (for $i = 1, 2, 3$). Such threshold effects can drastically correct the running behaviors of neutrino masses, flavor mixing angles and CP-violating phases, as already shown in Ref. [13]. Thus it makes sense to examine whether a constructive correction to the democratic lepton mixing ansatz can be obtained via the renormalization chain $\Lambda_{\rm GUT} \to M_3 \to M_2 \to M_1 \to \Lambda_{\rm EW}$. This is just the starting point of view of this work.

We propose a new phenomenological ansatz, in which the Yukawa coupling matrix of charged leptons results from the breaking of flavor democracy and the effective coupling matrix of light Majorana neutrinos is diagonal, at or close to the GUT scale. By using the one-loop renormalization-group equations (RGEs), we first calculate the radiative corrections to this ansatz and then confront it with current neutrino oscillation data at low-energy scales. To illustrate the RGE running and seesaw threshold effects, a simple but instructive numerical example will be presented. We demonstrate that it is possible to achieve the experimentally favored results for neutrino masses and lepton flavor mixing angles from our ansatz, provided the neutrino Yukawa coupling matrix takes a specific nontrivial pattern.

At the electroweak scale, the effective Lagrangian for lepton Yukawa interactions can be written as

$$-\mathcal{L} = \overline{E_L} H_1 Y_l l_R - \frac{1}{2} \overline{E_L} H_2 \cdot \kappa \cdot H_2^{c\dagger} E_L^c \ + \ {\rm h.c.} \tag{1}$$

[2]In the conventional seesaw mechanism [12] with three heavy right-handed neutrinos N_i (for $i = 1, 2, 3$), $\Lambda_{\rm SS}$ is sometimes referred to as M_1, the mass of the lightest $SU(2)_L$ singlet N_1. Without loss of generality, we take $M_3 > M_2 > M_1$ throughout this paper.

in the minimal supersymmetric standard model (MSSM) [3], where E_L denotes the leptonic $SU(2)_L$ doublets, H_1 and H_2 are the Higgs fields, l_R denotes the right-handed charged leptons, $H_2^c \equiv i\sigma^2 H_2^*$ and $E_L^c \equiv i\sigma^2 \mathcal{C}\overline{E_L}^T$ with $\mathcal{C}$ being the Dirac charge-conjugate matrix. After spontaneous gauge symmetry breaking, we arrive at the charged lepton mass matrix $M_l = vY_l \cos\beta$ and the effective Majorana neutrino mass matrix $M_\nu = v^2\kappa \sin^2\beta$, where $v \approx 174$ GeV and $\tan\beta$ is the ratio of the vacuum expectation values of H_2 and H_1 in the MSSM. The phenomenon of lepton flavor mixing, which arises from the mismatch between the diagonalization of Y_l and that of κ, is described by the 3×3 unitary matrix $V = V_l^\dagger V_\kappa$, where

$$
\begin{aligned}
V_l^\dagger (Y_l Y_l^\dagger) V_l &= \begin{pmatrix} y_e^2 & 0 & 0 \\ 0 & y_\mu^2 & 0 \\ 0 & 0 & y_\tau^2 \end{pmatrix} , \\
V_\kappa^\dagger \kappa V_\kappa^* &= \begin{pmatrix} \kappa_1 & 0 & 0 \\ 0 & \kappa_2 & 0 \\ 0 & 0 & \kappa_3 \end{pmatrix} .
\end{aligned} \tag{2}
$$

Of course, $m_\alpha = y_\alpha v \cos\beta$ (for $\alpha = e, \mu, \tau$) and $m_i = \kappa_i v^2 \sin^2\beta$ (for $i = 1, 2, 3$) are the masses of charged leptons and neutrinos, respectively. A very useful parametrization of V reads [14]

$$
V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -c_{12}s_{23}s_{13} - s_{12}c_{23}e^{-i\delta} & -s_{12}s_{23}s_{13} + c_{12}c_{23}e^{-i\delta} & s_{23}c_{13} \\ -c_{12}c_{23}s_{13} + s_{12}s_{23}e^{-i\delta} & -s_{12}c_{23}s_{13} - c_{12}s_{23}e^{-i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{i\rho} & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & 1 \end{pmatrix} , \tag{3}
$$

where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$ for $ij = 12, 23$ and 13. Current experimental data indicate $\theta_{12} \approx 33^\circ$, $\theta_{23} \approx 45^\circ$ and $\theta_{13} < 10^\circ$ [15], but there are not any constraints on the CP-violating phases δ, ρ and σ. Since δ governs the strength of CP violation in neutrino oscillations and has nothing to do with the neutrinoless double-beta decay, it is commonly referred to as the Dirac phase in contrast with the Majorana phases ρ and σ.

Above the seesaw scale, one encounters the neutrino Yukawa coupling matrix Y_ν together with the Majorana mass matrix M_R of three heavy right-handed neutrinos N_i:

$$
-\mathcal{L}' = \overline{E_L} H_2 Y_\nu N + \frac{1}{2}\overline{N^c} M_R N \ + \ \text{h.c.} \, . \tag{4}
$$

The seesaw mechanism [12] can naturally give rise to the effective neutrino coupling matrix $\kappa = Y_\nu M_R^{-1} Y_\nu^T$. Because three right-handed neutrinos are in general expected to have a mass hierarchy ($M_1 < M_2 < M_3$), however, it is necessary to take into account the seesaw threshold effects in the RGE running chain $\Lambda_{\rm GUT} \rightarrow M_3 \rightarrow M_2 \rightarrow M_1 \rightarrow \Lambda_{\rm EW}$ step by step (See Ref. [13] for a detailed description of how to treat the RGE running through each seesaw threshold). For simplicity, κ and V can empirically be extrapolated up to the GUT scale, where M_R can in turn be fixed by means of the inverted seesaw formula $M_R = Y_\nu^T \kappa^{-1} Y_\nu$. In this case, we may prescribe a phenomenological ansatz for the charged lepton Yukawa coupling matrix Y_l and the effective neutrino coupling matrix

[3] For the sake of simplicity, we assume the supersymmetry breaking scale $\Lambda_{\rm SUSY}$ to be close to the electroweak scale $\Lambda_{\rm EW}$. Even if $\Lambda_{\rm SUSY}/\Lambda_{\rm EW} \sim 10$, the relevant RGE running effects between these two scales are negligibly small for the physics under consideration [13].

κ at $\Lambda_{\rm GUT}$, and calculate radiative corrections to the lepton flavor mixing matrix V at $\Lambda_{\rm EW}$ by making use of the one-loop RGEs and by taking account of the seesaw threshold effects.

We propose that Y_l and κ take the following forms at $\Lambda_{\rm GUT}$:

$$
\begin{aligned}
Y_l &= \frac{c_l}{3}\left[\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} - \begin{pmatrix} \varepsilon_1 e^{i\phi_1} & 0 & 0 \\ 0 & \varepsilon_2 e^{i\phi_2} & 0 \\ 0 & 0 & \varepsilon_3 e^{i\phi_3} \end{pmatrix}\right] , \\
\kappa &= \begin{pmatrix} \kappa_1 e^{2i\varphi_1} & 0 & 0 \\ 0 & \kappa_2 e^{2i\varphi_2} & 0 \\ 0 & 0 & \kappa_3 e^{2i\varphi_3} \end{pmatrix} ,
\end{aligned}
\tag{5}
$$

where ε_i (for $i = 1, 2, 3$) are small perturbative parameters; i.e., $0 \leq \varepsilon_i \ll 1$. The role of ε_i is to break the flavor democracy of Y_l, such that the electron and muon masses can in turn be generated. The phase parameters ϕ_i of Y_l and φ_i of κ (for $i = 1, 2, 3$) will contribute, respectively, to the Dirac CP-violating phase δ and the Majorana CP-violating phases ρ and σ of V in Eq. (3). It is obvious that the unitary matrix V_κ defined to diagonalize κ in Eq. (2) is a pure phase matrix in our ansatz: $V_\kappa = {\rm Diag}\{e^{i\varphi_1}, e^{i\varphi_2}, e^{i\varphi_3}\}$. After Y_l is diagonalized by means of V_l, the lepton flavor mixing matrix $V = V_l^\dagger V_\kappa$ can then be calculated.

To illustrate, let us take a simple example by assuming $\varepsilon_1 = \varepsilon_2$, $\phi_1 = \phi_3 = 90^\circ$, $\phi_2 = 270^\circ$ and $\varphi_3 = 0^\circ$. As a result,

$$
c_l \approx y_\tau \; , \qquad \varepsilon_1 = \varepsilon_2 \approx \frac{3\sqrt{3 y_e y_\mu}}{y_\tau} \; , \qquad \varepsilon_3 \approx \frac{9 y_\mu}{2 y_\tau} \; . \tag{6}
$$

In a good approximation, we obtain the lepton flavor mixing matrix

$$
V \approx \begin{pmatrix} \dfrac{1-a}{\sqrt{2}} & \dfrac{1+a}{\sqrt{2}} & \sqrt{2}\, a \\ -\dfrac{1+3a}{\sqrt{6}} & \dfrac{1-3a}{\sqrt{6}} & \dfrac{2(1+ib)}{\sqrt{6}} \\ \dfrac{1}{\sqrt{3}} & -\dfrac{1}{\sqrt{3}} & \dfrac{1+ib}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} e^{i\varphi_1} & 0 & 0 \\ 0 & e^{i(\varphi_2 + 180^\circ)} & 0 \\ 0 & 0 & 1 \end{pmatrix} , \tag{7}
$$

where $a = \sqrt{y_e/(3y_\mu)}$ and $b = 3y_\mu/(2y_\tau)$. To compare between Eqs. (3) and (7), we redefine the phases of muon and tau fields: $\mu \to \mu e^{ib}$ and $\tau \to \tau e^{ib}$. Then the expression of V in Eq. (7) can approximately be transformed into

$$
V \approx \begin{pmatrix} \dfrac{1-a}{\sqrt{2}} & \dfrac{1+a}{\sqrt{2}} & \sqrt{2}\, a \\ -\dfrac{1+3a}{\sqrt{6}} e^{-ib} & \dfrac{1-3a}{\sqrt{6}} e^{-ib} & \dfrac{2}{\sqrt{6}} \\ \dfrac{1}{\sqrt{3}} e^{-ib} & -\dfrac{1}{\sqrt{3}} e^{-ib} & \dfrac{1}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} e^{i\varphi_1} & 0 & 0 \\ 0 & e^{i(\varphi_2 + 180^\circ)} & 0 \\ 0 & 0 & 1 \end{pmatrix} , \tag{8}
$$

up to the accuracy of $\mathcal{O}(a)$ and $\mathcal{O}(b)$. It becomes obvious that Eq. (8) is a reasonable simplification of Eq. (3) by neglecting the small $\mathcal{O}(s_{13})$ terms from its $V_{\mu 1}$, $V_{\mu 2}$, $V_{\tau 1}$ and

$V_{\tau 2}$ elements. We are therefore left with $\delta \approx b$, $\rho \approx \varphi_1$ and $\sigma \approx \varphi_2 + 180^\circ$. Of course, the values of all relevant parameters appearing in Eqs. (6), (7) and (8) are set at $\Lambda_{\rm GUT}$.

To run the results obtained at $\Lambda_{\rm GUT}$ to the electroweak scale by using the one-loop RGEs [13], it is necessary to fix the neutrino Yukawa coupling matrix Y_ν. Corresponding to Eq. (5), a convenient parametrization of Y_ν can be taken as

$$Y_\nu = y_\nu U_\nu \begin{pmatrix} r_1 r_2 & 0 & 0 \\ 0 & r_2 & 0 \\ 0 & 0 & 1 \end{pmatrix} , \tag{9}$$

where y_ν, r_1 and r_2 are three real and positive dimensionless parameters characterizing the eigenvalues of Y_ν, and

$$U_\nu = \begin{pmatrix} e^{i\xi} & 0 & 0 \\ 0 & e^{i\zeta} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_1 c_3 & s_1 c_3 & s_3 \\ -c_1 s_2 s_3 - s_1 c_2 e^{-i\omega} & -s_1 s_2 s_3 + c_1 c_2 e^{-i\omega} & s_2 c_3 \\ -c_1 c_2 s_3 + s_1 s_2 e^{-i\omega} & -s_1 c_2 s_3 - c_1 s_2 e^{-i\omega} & c_2 c_3 \end{pmatrix} \tag{10}$$

with $c_i \equiv \cos\theta_i$ and $s_i \equiv \sin\theta_i$ (for $i = 1, 2, 3$). Because Y_ν totally involves nine free parameters, there will be much freedom in adjusting the RGE running behaviors of y_α (for $\alpha = e, \mu, \tau$), κ_i (for $i = 1, 2, 3$) and V to fit current experimental data. This category of uncertainties is likely to be more or less reduced in a unified model of leptons and quarks (e.g., the $SO(10)$ model [16]), in which the texture of Y_ν could be related to that of quarks. Guided by the principle of simplicity and naturalness, we shall try to pick on a reasonable parameter space of Y_ν by avoiding possible fine-tuning of the input parameters in our numerical calculations.

Now we present a numerical example to illustrate the RGE corrections to the results obtained in Eqs. (6) and (8). The eigenvalues of Y_l at $\Lambda_{\rm GUT}$ (i.e. y_e, y_μ and y_τ) are chosen in such a way that they can correctly run to their low-energy values [11]. Then the initial values of three mixing angles (θ_{12}, θ_{23} and θ_{13}) and the Dirac phase (δ) of V can largely be determined via Eq. (8). The mass spectrum of three light neutrinos is assumed to have a near degeneracy with $m_1 \approx 0.245$ eV, $\Delta m^2_{21} \equiv m^2_2 - m^2_1 > 0$ and $\Delta m^2_{31} \equiv m^2_3 - m^2_1 > 0$ at $\Lambda_{\rm GUT}$. Thus the initial values of κ_1, κ_2 and κ_3 can be chosen by using

$$\begin{aligned} \kappa_1 &= \frac{m_1}{v^2 \sin^2\beta} , \\ \kappa_2 &= \frac{\sqrt{m^2_1 + \Delta m^2_{21}}}{v^2 \sin^2\beta} , \\ \kappa_3 &= \frac{\sqrt{m^2_1 + \Delta m^2_{31}}}{v^2 \sin^2\beta} , \end{aligned} \tag{11}$$

together with a typical input $\tan\beta = 10$, such that the resultant neutrino mass-squared differences at $\Lambda_{\rm EW}$ are consistent with current solar and atmospheric neutrino oscillation data [1]–[4]. Furthermore, we assume that the eigenvalues of Y_ν are strongly hierarchical (i.e., $0 < r_1 \ll 1$ and $0 < r_2 \ll 1$, just like the case of quarks) and $y_\nu \sim \mathcal{O}(1)$ holds. It turns out that only θ_1, θ_2, ξ and ζ of U_ν are important for the RGE running behaviors of V. We find that θ_3 and ω of U_ν may contribute a little to the renormalization effects on V, only when r_1 and r_2 are not so small. A summary of the input values of relevant

parameters for radiative corrections to our phenomenological ansatz is given in Table 1, where the outputs of $(m_1, \Delta m^2_{21}, \Delta m^2_{31})$, $(\theta_{12}, \theta_{23}, \theta_{13})$ and (δ, ρ, σ) at $\Lambda_{\rm EW}$ are also listed.

One can see that it is essentially possible to reproduce the best-fit values of Δm^2_{21}, Δm^2_{31}, θ_{12} and θ_{23}, which are already determined from a global analysis of current experimental data on solar and atmospheric neutrino oscillations [15], from our lepton mass matrices proposed at $\Lambda_{\rm GUT}$. The output $\theta_{13} \approx 7.7^\circ$ is acceptable, because it is in no conflict with the upper limit $\theta_{13} < 10^\circ$ set by the global fit together with the CHOOZ experiment [17] at the 99% confidence level. The output $m_1 \approx 0.2$ eV implies that the masses of three light Majorana neutrinos are nearly degenerate at $\Lambda_{\rm EW}$, and the effective mass of the neutrinoless double-beta decay is of the same order (i.e., $\langle m\rangle_{ee} \approx m_1$). The latter is certainly consistent with the present experimental upper bound $\langle m\rangle_{ee} < 0.38$ eV at the 99% confidence level [15]. We plot the running behaviors of m_1, Δm^2_{21} and Δm^2_{31} in Fig. 1(a) and those of θ_{12}, θ_{23} and θ_{13} in Fig. 1(b), from which the seesaw threshold effects can clearly be seen. Indeed, the inverted seesaw relation $M_R = Y^T_\nu \kappa^{-1} Y_\nu$ yields $M_1 \approx 4.1 \times 10^8$ GeV, $M_2 \approx 1.6 \times 10^{11}$ GeV and $M_3 \approx 7.2 \times 10^{13}$ GeV in our ansatz. Therefore, the most significant radiative corrections to both neutrino masses and lepton flavor mixing angles appear in the region between M_3 and $\Lambda_{\rm GUT}$. In other words, it is the seesaw threshold M_3 that can remarkably change the RGE evolution of relevant physical parameters, such that the ansatz of democratic lepton mixing becomes viable to fit today's low-energy neutrino data.

Finally, let us make some further remarks.

(1) It is worth mentioning that one may also discuss the phenomenological scenario in Eq. (5) and the relevant RGE running effects beyond the numerical example taken above. Similar results can then be anticipated at $\Lambda_{\rm EW}$. The reason is simply that the perturbative matrix in Y_l does not affect the dominant part of V, which may arise from any slight breaking of the flavor democracy of Y_l.

(2) We have made use of the complicated technique developed in Refs. [9, 13] to deal with the RGE running and seesaw threshold effects, but our phenomenological ansatz and its low-energy consequences are new and irrelevant to the numerical examples taken in Ref. [13]. The simple but typical analysis given by us should be more suggestive and useful for model building, in particular in the spirit of flavor democracy and its explicit breaking at a superhigh energy scale.

(3) A number of assumptions have been taken in our treatment of radiative corrections. Some comments on them are in order.

- *The strong mass hierarchy of three heavy right-handed neutrinos.* This assumption, which seems to be more natural than the assumption of three degenerate (or partially degenerate) right-handed neutrinos, is mainly to illustrate the seesaw threshold effects in a more general and convincing way. Our numerical results have shown that it is the RGE running between $\Lambda_{\rm GUT}$ and M_3 that plays the dominant role in the renormalization chain $\Lambda_{\rm GUT} \to M_3 \to M_2 \to M_1$. Such an interesting feature, which is essentially not subject to a specific model or ansatz at the GUT scale, has already been observed in Ref. [13].

- *The strong hierarchy of three eigenvalues of Y_ν.* This assumption may become quite reasonable, if our ansatz is embedded in the $SO(10)$ models [16], in which the texture of Y_ν can be related to that of quarks with a strong mass hierarchy. It technically

simplifies our numerical analysis, because it forbids a few free parameters of Y_ν to play an important role in controlling the RGE evolution of V.

- *The approximate mass degeneracy of three light neutrinos.* This assumption will be rather meaningful, provided the $S(3)$ symmetry is imposed on the effective neutrino coupling matrix κ as a starting point of view of model building [6, 7]. It is also a crucial prerequisite to give rise to significant RGE running effects, such that the ansatz proposed at $\Lambda_{\rm GUT}$ can be compatible with current neutrino oscillation data obtained at low energies. If the masses of three light neutrinos are taken to be hierarchical, however, the democratic lepton mixing ansatz under consideration will not be viable in phenomenology.

The above discussions indicate that the parameter space considered in our analysis is actually typical and instructive. Furthermore, the freedom in adjusting those model parameters can be restricted to a certain extent both by some theoretical arguments and by the experimental data.

In summary, we have proposed a new ansatz of democratic lepton mixing at the GUT scale and examined possible radiative corrections to its phenomenological consequences. We show that it is possible to obtain the experimentally favored results for both the neutrino mass spectrum and the lepton flavor mixing angles from this ansatz, provided the neutrino Yukawa coupling matrix takes a specific nontrivial pattern. The seesaw threshold effects are found to play a significant role in the RGE running of relevant physical quantities.

Although the numerical example presented in this paper is mainly for the purpose of illustration, it is quite suggestive for model building. We believe that the breaking of lepton flavor democracy at a superhigh energy scale is an interesting phenomenological approach towards deeper understanding of the bi-large mixing pattern of lepton flavors observed in the solar and atmospheric neutrino oscillation experiments, and it might even hint at the underlying flavor dynamics which governs the generation of fermion masses and the origin of CP violation.

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Table 1: A numerical example for radiative corrections to the proposed ansatz of lepton mass matrices (from $\Lambda_{\rm GUT}$ to $\Lambda_{\rm EW}$) in the MSSM, where $\tan\beta = 10$ has typically been taken.

Parameter	Input ($\Lambda_{\rm GUT}$)	Output ($\Lambda_{\rm EW}$)
m_1(eV)	0.245	0.2
$\Delta m^2_{21}(10^{-5}\ {\rm eV}^2)$	25	7.8
$\Delta m^2_{31}(10^{-3}\ {\rm eV}^2)$	9	2.2
θ_{12}	47.2°	33.6°
θ_{23}	54.4°	45.3°
θ_{13}	3.1°	7.7°
δ	5.0°	96.6°
ρ	62.6°	29°
σ	355.4°	302.8°
y_ν	0.87	
r_1	0.048	
r_2	0.042	
θ_1	36°	
θ_2	11°	
θ_3	10°	
ω	252°	
ξ	331.5°	
ζ	143°	
φ_1	62°	
φ_2	176°	

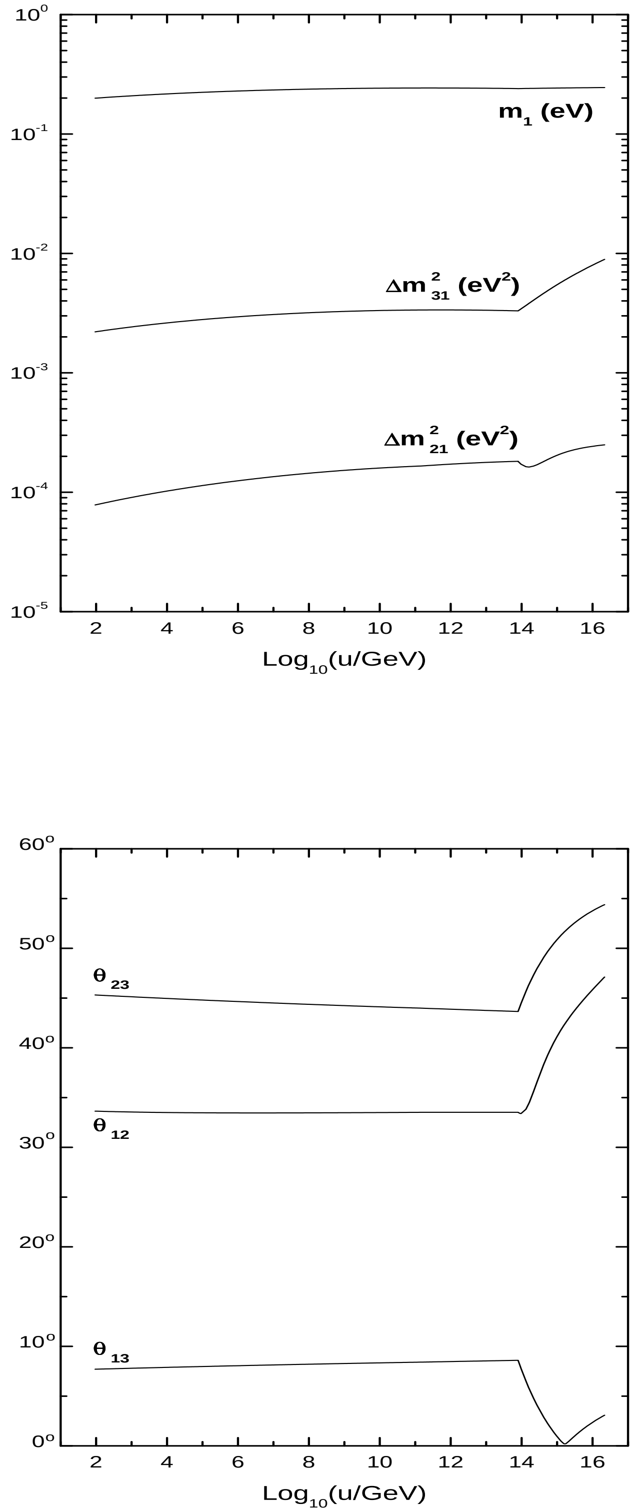

Figure 1: (a) the running behaviors of m_1, Δm^2_{21} and Δm^2_{31} between $\Lambda_{\rm EW}$ and $\Lambda_{\rm GUT}$; (b) the running behaviors of θ_{12}, θ_{23} and θ_{13} between $\Lambda_{\rm EW}$ and $\Lambda_{\rm GUT}$. The input values of relevant parameters are listed in Table 1.